11. V. I. Zharkov and V. A. Kalinin, Equations of State of Solids at High Pressures and Temperatures, Nauka, Moscow (1968).

EFFECT OF NONEQUILIBRIUM HEATING ON THE BEHAVIOR OF A POROUS MATERIAL IN SHOCK COMPRESSION

A. V. Attetkov, L. N. Vlasova,
V. V. Selivanov, and V. S. Solov'ev

UDC 534.222.2+662.215.4

The propagation of low-intensity shock waves (of the order of several gigapascals) in porous solids has a number of particularities [1] associated with the nonholonomicity of the equation of state [2]. The behavioral characteristics of a porous material on a range of pressures comparable with its strength are related to the irreversible nature of the deformation of the medium and the importance of the influence of its strength and viscous properties [2]. In [3] it was shown that the anomalous behavior of the shock adiabat cannot be explained starting from the assumption that the porous material is in a state of thermodynamic equilibrium under shock compression. The main cause of the absence of thermodynamic equilibrium behind the shock front is thought to be the nonuniform heating of the material, which is confirmed by experiment [4-6]. The results of [7, 8] indicate that heating of the porous material is most pronounced in the neighborhood of the pores, where the temperature may exceed the melting points.

Theoretical studies [9] have made it possible to establish the characteristics of the behavior of a porous substance under shock compression associated with local melting in the vicinity of the pores. It has been shown that the reduction in mechanical strength due to local melting leads to a break in the shock adiabat at the point at which melting begins. In this case the nonuniformity in the distribution of the dissipated energy depends to a considerable extent on the initial porosity. The investigation [9] was carried out on the assumption that the characteristic pore collapse times are substantially less than the characteristic thermal relaxation times.

Experimental studies [10-12] indicate that the initial pore size has a considerable influence on the nonequilibrium character of the heat release in shock compression, especially in the region of low loading pressures. Similar conclusions were reached in investigating the effect of particle size on the sensitivity of explosive charges to ignition [13-15].

Below we investigate the effect of initial pore size on the heating of a material in shock compression. The influence of the shock wave amplitude, the viscosity coefficient, and the yield point on the maximum possible temperature rise is analyzed. The dependence of the heating dynamics on the thermophysical properties of the porous material is studied.

Let us consider the behavior of a porous material in response to the propagation of a low-intensity shock wave whose amplitude is so small that the compressibility of the solid can be neglected, but large enough for viscoplastic flow to develop in the vicinity of the pores. On this pressure interval the width of the shock front is much greater than the size of the inhomogeneities [1], and the change in density is mainly attributable to the collapse of the pores. We will base our investigation on the spherical cell model [1, 2], assuming that on the pressure range in question the density of the solid ρ_s is constant. The porosity parameter, or the ratio of the total volume of the material to the solid volume, is $\alpha = b^3/(b^3 - a^3)$, where b and a are the instantaneous radii of the cell and the pore. The initial cell radius b_0 is found from the condition that the total mass of the cells per unit mass is equal to 1, i.e., $4\pi N\rho_s(b_0^3 - a_0^3)/3 = 1$, where N is the number of cells per unit mass. Using the definition for the initial porosity parameter α_0, we can also write this condition in the form

$$4\pi N\rho_s a_0^3/[3(\alpha_0 - 1)] = 1.$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 120-127, November-December, 1984. Original article submitted October 10, 1983.

 0021-8944/84/2506-0914$08.50

Consequently, a change in the initial pore size a_0 in a material of given porosity α_0 is equivalent to a change in the total number of cells (pores) per unit mass of material.

In the coordinate system tied to the wave, the amplitude p and the velocity D of the shock wave are determined by the integral laws of conservation at the shock front which, with allowance for the relation $\rho = \rho_s/\alpha$ (it is assumed that the amount of gas in the pores is insignificant), lead to the expression [1]

$$p - p_0 = \rho_s D^2 (\alpha_0 - \alpha)/\alpha_0^2, \tag{1}$$

where ρ is the density of the porous medium; $p_* = -(2Y/3)\ln[(\alpha_0 - 1)/\alpha_0]$ is the amplitude of the elastic precursor. In this case D must exceed a certain value $D_0 = \{2Y\alpha_0/[3\rho_s(\alpha_0 - 1)]\}^{1/2}$, characterizing the minimum velocity of propagation of a stationary shock wave in a porous medium [2].

The functional relation between the average pressure p in the porous medium and the porosity parameter α in Eq. (1) is determined from the solution of the dynamic problem of the deformation of the solid-phase material in the vicinity of a pore. For spherically symmetrical compression of the cell, in the coordinate system tied to the center of the pore, when there is only radial motion and all the parameters depend on the Euler coordinate r and time t, the continuity equation and the equation of motion take the form

$$\partial\rho_s/\partial t + r^{-2}\partial(\rho_s r^2 v)/\partial r = 0; \tag{2}$$

$$\rho_s(\partial v/\partial t + v\partial v/\partial r) = \partial\sigma_r/\partial r + (2/r)(\sigma_r - \sigma_\theta). \tag{3}$$

On the pressure range investigated the solid-phase material complies with the relations for a viscoplastic medium

$$\sigma_r - \sigma_\theta = Y + 2\eta(\partial v/\partial r - v/r). \tag{4}$$

The boundary condition at the pore surface is written in the form

$$\sigma_r|_{r=a} = 0. \tag{5}$$

Here, v is the radial velocity σ_r and $\sigma_\theta = \sigma_\varphi$ are the stress tensor components; Y and η are the yield stress and the coefficient of viscosity of the solid.

Using the assumption of incompressibility of the solid-phase material (ρ_s = const), we determine the first and second integrals of Eq. (2) in the form

$$v = \dot{a}(a/r)^2, \quad r^3 - r_0^3 = a^3 - a_0^3, \tag{6}$$

where a dot denotes differentiation with respect to time; the zero subscript signifies initial values.

Taking into account the fact that the pressure in the solid phase $p_s = -(\sigma_r + 2\sigma_\theta)/3$, and using (4) and (6), we reduce Eq. (3) to the form

$$\rho_s(\partial v/\partial t + v\partial v/\partial r) = -\partial p_s/\partial r + 2Y/r. \tag{7}$$

In this case the effect of viscosity on the process will be expressed only through the boundary condition which, using (6), we write in the form

$$p_s|_{r=a} = 2Y/3 - 4\eta\dot{a}/a. \tag{8}$$

Integrating (7) over the radius from a to r and using (6) and (8), we obtain the pressure distribution $p_s(r, t)$ in the vicinity of the pore, averaging which over the volume of the spherical cell makes it possible to determine the average macroscopic pressure in the solid phase of the porous material:

$$p_1 = -\rho_s\left[a(1-\varphi_1)\ddot{a} + \frac{3}{2}(1-\varphi_2)\dot{a}^2\right] - \frac{4\eta\dot{a}}{a} - \frac{2Y\alpha}{3}\ln\frac{(\alpha-1)}{\alpha}, \tag{9}$$

where

$$\varphi_1 = \frac{3(m^{1/3} - m)}{2(1-m)}, \quad \varphi_2 = \frac{m^{1/3}(2+m) - 3m}{(1-m)}, \quad m = \frac{(\alpha-1)}{\alpha}.$$

It is easy to show that for the model in question the average pressure p in the porous medium is related to the average macroscopic pressure p_1 in the solid phase of the material by the expression

$$p = p_1/\alpha. \tag{10}$$

The joint solution of Eqs. (9) and (10) makes it possible to determine the nature of the motion of the material in the vicinity of the pores in terms of the average pressure in the medium. On going over from the variables (a, t) to the variables (α, t) the system of equations (9), (10) reduces to the equation $p = p(\ddot{\alpha}, \dot{\alpha}, \alpha)$ obtained in [2].

On collapse of the pores, the local heating associated with the competition between the processes of heat release due to plastic and viscous friction and removal of heat by conduction is determined from the solution of the following system of equations

$$\varsigma_s \rho_s\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial r}\right) = \frac{\lambda_s}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{2}{3}(\sigma_r - \sigma_\theta)\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right),$$
$$r(0) = r_0,\ T(0, r) = T_0,\ v(0, r) = 0, \tag{11}$$
$$\partial T/\partial r|_{r=a} = 0,\ \partial T/\partial r|_{r=b} = 0,$$

where c_s, λ_s, T are the specific heat, thermal conductivity, and temperature of the solid.

We introduce the dimensionless variables and parameters:

$$\tau = \frac{t}{a_0}\left(\frac{p}{\rho_s}\right)^{1/2},\quad R = \frac{r}{a_0},\quad R_1 = \frac{a}{a_0},\quad R_2 = \frac{b}{a_0},\quad w_1 = \dot{a}\left(\frac{p}{\rho_s}\right)^{-1/2},$$
$$\beta = \frac{Y}{p},\quad \mathrm{Re} = \frac{a_0\sqrt{p\rho_s}}{\eta},\quad \mathrm{Pr} = \frac{\varsigma_s\eta}{\lambda_s} = \frac{\eta}{\varkappa_s\rho_s},\quad \Theta = \frac{\varsigma_s\rho_s}{p}(T - T_0).$$

Here, $\varkappa_s$ is the thermal diffusivity of the solid; w_1 is the velocity of the pore boundary; the subscripts 1 and 2 denote the coordinates of the pore radius and the radius of the spherical cell. The parameter β characterizes the plastic properties of the material, the Reynolds number Re its viscous properties, and the Prandtl number Pr the relation between the amount of heat released as a result of viscous friction and the amount of heat removed by conduction.

In the new variables, using (4) and (6), we can reduce Eq. (11), describing the nonstationary heating of the material in the vicinity of a pore, to the form

$$\frac{\partial\Theta}{\partial\tau} + w_1\left(\frac{R_1}{R}\right)^2\frac{\partial\Theta}{\partial R} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}\,R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial\Theta}{\partial R}\right) - 2\beta\frac{w_1R_1^2}{R^3} + \frac{12}{\mathrm{Re}}\frac{w_1^2R_1^4}{R^6}, \tag{12}$$

and the law of variation of the pore radius R_1 and its velocity w_1 are determined from the solution of the following system of differential equations:

$$(1 - \varphi_1)\frac{dw_1}{d\tau} = -\frac{3(1 - \varphi_2)}{2R_1}w_1^2 - \frac{4w_1}{\mathrm{Re}\,R_1^2} - \frac{\alpha}{R_1}\left[1 + \frac{2\beta}{3}\ln\frac{(\alpha - 1)}{\alpha}\right], \tag{13}$$
$$dR_1/d\tau = w_1,$$

where the coefficients φ_1 and φ_2 are determined in accordance with (9).

The initial and boundary conditions take the form

$$R_1(0) = 1,\ w_1(0) = 0,\ \Theta(0, R) = 0,$$
$$\partial\Theta/\partial R|_{R=R_1} = 0,\ \partial\Theta/\partial R|_{R=R_2} = 0. \tag{14}$$

In the new variables the macroscopic porosity parameter is given by the expression

$$\alpha = 1 + R_1^3(\alpha_0 - 1). \tag{15}$$

The last two terms in Eq. (12) reflect the intensity of the heat release due to plastic and viscous friction respectively.

In view of the irreversibility of the loading-unloading diagram [16, 17], system of equations (13) is applicable only in the loading phase ($w_1 \leqslant 0$, $0 \leqslant R_1 \leqslant 1$), since during unloading it is necessary to take into account the elastic or elastoplastic properties of the medium.

Thus the analysis of the local heating dynamics of a porous material under shock compression reduces to the solution of system of equations (12), (13), (15) with initial and boundary conditions (14). The form of the function $\Theta(r, R)$ is determined by the set of independent parameters (α_0, β, Re, and Pr); in this case the parameter β is bounded by the quantity β_0, which characterizes the condition of transition to the plastic state:

$$\beta \leqslant \beta_0 \equiv -(3/2)[\ln(\alpha_0 - 1)/\alpha_0]^{-1}.$$

An analysis of Eq. (12) shows that the highest temperatures occur in the vicinity of a pore. We will determine the maximum possible temperature rise attainable on the assumption that there are no heat conduction processes (this corresponds to the condition $\mathrm{Pr} \to \infty$). Going over in (12) from the variable τ to the variable R_1 (as a result of the monotonicity of the function $R_1(\tau)$ in the loading phase) and taking into account boundary conditions (14) at $R = R_1$, we obtain

$$\Theta_1(R_1) = -2\beta \ln R_1 + \frac{12}{\mathrm{Re}} \int_1^{R_1} \frac{w_1(R_1')}{R_1'^2}\, dR_1', \tag{16}$$

where the first and second terms on the right of the expression represent the heating due to plastic and viscous friction respectively.

At $R_1 \to 0$ ($\alpha \to 1$) the plasticity term has a logarithmic singularity, and the viscous friction term tends to infinity as $R_1^{-5/2}$, since on collapse of the pore $w_1 \sim R_1^{-3/2}$ [2]. A similar result was obtained in [9] starting from a consideration of the energy accumulation in the neighborhood of the pores. In fact, the heating described by Eq. (16) is bounded by the melting point of the medium; when this is reached the material near the pore goes over into the liquid state and the parameter vanishes, since $Y = 0$.

In the limiting case $\mathrm{Re} \ll 1$ (this corresponds to either large values of the viscosity coefficient of the solid or small values of the initial pore radius a_0), neglecting the inertia terms in (13), we obtain an explicit form of the expression for the pore boundary velocity:

$$w_1(R_1) = -\frac{\mathrm{Re}\, R_1}{4}\left[1 + R_1^3(\alpha_0 - 1)\right]\left\{1 - \frac{2\beta}{3}\ln\left[1 + R_1^{-3}(\alpha_0 - 1)^{-1}\right]\right\}. \tag{17}$$

The law of motion of the pore boundary given by the expression $\tau = \int_1^{R_1} dR_1'/w_1(R_1')$ can be reduced, using (17), to the relation

$$\tau = \frac{2}{\beta\,\mathrm{Re}} \ln\left[\frac{1 + (2\beta/3)\ln\left(1 - \alpha_0^{-1}\right)}{1 - (2\beta/3)\ln\left[1 + R_1^{-3}(\alpha_0 - 1)^{-1}\right]}\right]. \tag{18}$$

Substituting (17) in (16) and carrying out the integration, we obtain the following relations:

$$\Theta_1(R_1) = F(R_1) - \frac{2\beta}{3}\int_{y_0}^{y} \frac{\ln(1 + y')}{y'}\, dy',$$

$$F(R_1) = -2\beta \ln R_1 + (\alpha_0 - 1)(1 - R_1^3) - 3\ln R_1 - 2\beta R_1^3(\alpha_0 - 1)\ln R_1 + \\ + (2\beta/3)\{[1 + R_1^3(\alpha_0 - 1)]\ln[R_1^3 + (\alpha_0 - 1)^{-1}] + \alpha_0 \ln(1 - \alpha_0^{-1})\},$$

where

$$y = R_1^{-3}(\alpha_0 - 1)^{-1}.$$

The last integral can be expressed in terms of elementary functions:

$$\int_{y_0}^{y} \frac{\ln(1 + y')}{y'}\, dy' = \frac{3}{2}\ln R_1 [2\ln(\alpha_0 - 1) + 3\ln R_1] - \sum_{l=1}^{n} (-1)^l \frac{(\alpha_0 - 1)^l\left(1 - R_l^{3l}\right)}{l^2}.$$

At small values of the initial porosity α_0, confining ourselves to the first term of the series, after transformations we obtain

$$\Theta_1(R_1) = (\alpha_0 - 1)(1 - R_1^3)(1 - 2\beta/3) + (2\beta/3)\{[1 + R_1^3(\alpha_0 - 1)]\times \\ \times \ln[R_1^3 + (\alpha_0 - 1)^{-1}] + \alpha_0 \ln(1 - \alpha_0^{-1})\} - 2\beta \ln R_1 [\ln(\alpha_0 - 1) + \\ + R_1^3(\alpha_0 - 1) + 1] - 3\ln R_1 (1 + \beta \ln R_1).$$

From the expressions obtained it follows that starting from a certain pore radius the maximum possible temperature rise for a material with porosity α_0 but different initial sizes of the inhomogeneities does not depend on the quantity a_0 and is determined solely by the plastic properties of the material.

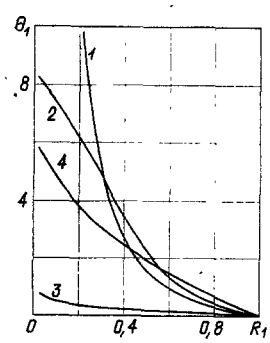

Fig. 1

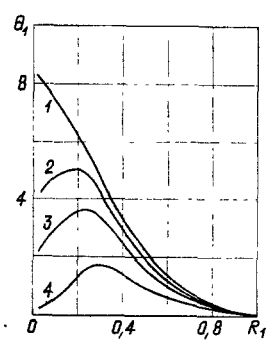

Fig. 2

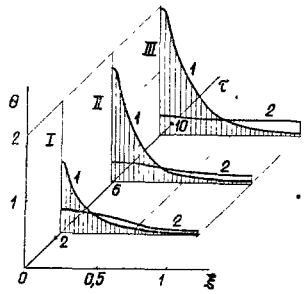

Fig. 3

According to the literature (see, for example, [18-20]), for most metals the value of the viscosity coefficient is $(10^3$-$10^4)$ Pa·sec. Taking $\rho_s = 10^4$ kg/m^3, we find that for shock wave loading on the pressure interval 1-10 GPa the Reynolds parameter takes values of 0.03-1.0 at $a_0 = 100$ μm and 0.003-0.1 at $a_0 = 10$ μm. Thus, the condition Re << 1 holds over a broad range of variation of the values of a_0.

The results of a numerical solution of Eqs. (13) and (16) are presented in Fig. 1. Curves 1-3 illustrate the nature of the dependence $\Theta_1(R_1)$ at $\alpha_0 = 1.11$, $\beta = 0.1$ and the following values of the Re parameter: 1, 10; 2, 5; 3, ∞. Curve 4 gives the value of the dimensionless temperature for the case Re << 1. A decrease in Reynolds number (equivalent to a decrease in a_0) leads to a change in the nature of the increase in Θ_1; in this case the temperature rise is considerable even in the initial stages of the pore collapse process. The effect of the plastic properties of the medium, characterized by the parameter β, on the quantity Θ_1 at fixed values of the parameters (α_0, Re) reduces to a decrease in dimensionless temperature with decrease in β.

In [2, 9] attention was drawn to the possibility of pore collapse at relatively low shock wave amplitudes (of the order of 1 GPa) as a result of local melting and the inertial motion of the material towards the center of the cavities. The investigations show that the behavioral characteristics of a shocked porous medium due to local melting in the vicinity of the pores depend to a considerable extent on the initial pore size and are expressed on a narrow range of variation of a_0. This conclusion is based on the fact that at large Reynolds numbers (corresponding to large values of a_0) if pore collapse occurs, then it is due to the inertial motion of the material towards the center of the cavities, since in this case the effect of local temperature gradients is expressed only in the final stages of the pore collapse process. As the Re parameter decreases, the conditions of formation of a local melting zone in the vicinity of the pores may develop even in the initial stages of the pore collapse process, but in this case thermal condition effects will exert an ever-increasing influence on the end value of the temperature. Under certain conditions the rate of heat removal exceeds the rate of heat release in plastic and viscous friction even in the initial stages of the process, as a result of which the material in the vicinity of the pores will remain practically unheated.

In order to investigate the effect of thermal conduction on the heating dynamics of a porous material under shock compression, we carried out a numerical integration of system of equations (12)-(15). The solution of the problem is obtained by going over to a moving coordinate system using the variable $\xi = (R - R_1)/(R_2 - R_1)$, $\xi \in [0, 1]$.

Using the conversion formulas

$$\frac{\partial}{\partial R} = \frac{1}{(R_2 - R_1)} \frac{\partial}{\partial \xi}, \quad \left(\frac{\partial}{\partial \tau}\right)_R = \left(\frac{\partial}{\partial \tau}\right)_\xi + \frac{[\xi(\dot{R}_1 - \dot{R}_2) - \dot{R}_1]}{(R_2 - R_1)} \frac{\partial}{\partial \xi}$$

and taking into account the fact that $\dot{R}_1 = w_1$ and $\dot{R}_2 = w_1(R_1/R_2)^2$, we reduce Eq. (12) to the form

$$\frac{\partial \Theta}{\partial \tau} + f_1(\tau, \xi) \frac{\partial \Theta}{\partial \xi} = f_2(\tau) \frac{\partial^2 \Theta}{\partial \xi^2} + f_3(\tau, \xi), \tag{19}$$

where

$$f_1(\tau, \xi) = \frac{1}{(R_2 - R_1)} \left[w_1 \left\{ \xi \left[1 - \left(\frac{R_1}{R_2} \right)^2 \right] + \left(\frac{R_1}{R} \right)^2 - 1 \right\} - \frac{2}{\mathrm{Pr\,Re}\, R} \right],$$

$$f_2(\tau) = [\mathrm{Pr\,Re}\,(R_2 - R_1)^2]^{-1},$$

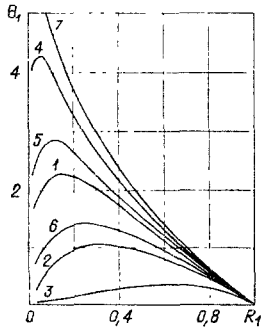

Fig. 4

$$f_3(\tau, \xi) = -2\beta w_1 R_1^2 R^{-3} + (12/\mathrm{Re})\, w_1^2 R_1^4 R^{-6},$$
$$R(\xi) = R_1 + \xi(R_2 - R_1).$$

The initial and boundary conditions are written in the form

$$R_1(0) = 1, \quad R_2(0) = (1 - \alpha_0^{-1})^{-1/3}, \quad w_1(0) = 0, \quad \Theta(0, \xi) = 0,$$
$$\partial\Theta/\partial\xi|_{\xi=0} = 0, \ \partial\Theta/\partial\xi|_{\xi=1} = 0.$$

In order to determine the temperature profile $\Theta(\tau, \xi)$ the following calculation method is used. For each time step $\Delta\tau$ from system of equations (13) we calculate the functions $R_1(\tau)$, $w_1(\tau)$, and $R_2(\tau) = R_1[1 + R_1^{-3}(\alpha_0 - 1)^{-1}]^{1/3}$, whose values are substituted in (19). By means of an implicit symmetrical scheme the latter equation is approximated by a system of algebraic equations solved for each time step by the pivotal method. Over the entire region of variation of the parameters (except for the region $\mathrm{Re} \ll 1$) the step $\Delta\tau$ was taken equal to 10^{-2}. A numerical experiment showed that the required accuracy is achieved on satisfaction of the condition $\Delta\tau/(\mathrm{PrRe}\, h^2) \geqslant 10^3$, where h is the step with respect to the space variable. In the region of values of the Reynolds number satisfying the condition $\mathrm{Re} \ll 1$, for determining the values of the functions $R_1(\tau)$, $w_1(\tau)$ we used the approximate expressions (17) and (18).

The influence of thermal conduction on the heating of the porous medium is characterized by the value of the Prandtl number. The curves in Fig. 2 illustrate the variation of the dimensionless pore surface temperature at the following values of the Prandtl number: 1, ∞; 2, 200; 3, 20; 4, 2; the other parameters were fixed: $\alpha_0 = 1.11$; $b = 0.1$; $\mathrm{Re} = 5$. A decrease in the Prandtl number (equivalent to an increase in the thermal diffusivity $\varkappa_s$) leads to a decrease in the end (highest) values of the temperature.

The competition between the processes of heat release due to plastic and viscous friction and heat removal as a result of thermal conduction is illustrated by the temperature profiles $\Theta(\tau, \xi)$ in Fig. 3. The calculations were made for $\alpha_0 = 1.11$, $\beta = 0.1$, and $\mathrm{Re} = 1$. The figures I-III correspond to values of the pore radius $R_1 = 0.6$, 0.32, and 0.2. At $\mathrm{Pr} = 10^3$ (curve 1) the rate of heat release predominates, which leads to characteristic temperature gradients in the vicinity of the pores and hence to essentially nonuniform heating of the porous material. The temperature profile for $\mathrm{Pr} = 10$ (curve 2) corresponds to the case where the rate of heat removal predominates over the rate of heat release even in the initial stages of the pore collapse process. The results obtained point to the important influence of the Prandtl number on the developing temperature profile and hence on the time required for thermodynamic equilibrium to be established in the shocked material.

As the Reynolds number decreases, the effect of thermal conduction on the heating dynamics and the end value of the temperature increases. On the other hand, the investigations show that the maximum possible temperature rise for a material with porosity α_0 at a fixed value of the parameter β is limited by the temperature value Θ_1 corresponding to the case $\mathrm{Re} \ll 1$. The effect of the Prandtl number on the variation of the pore surface temperature in the region $\mathrm{Re} \ll 1$ is illustrated in Fig. 4. Curves 1-3 correspond to $\mathrm{Re} = 0.1$, curves 4-6 to $\mathrm{Re} = 0.5$. The Pr parameter was taken equal to 10^5 (curves 1, 4), 10^4 (curves 2, 5), and 10^3 (curves 3, 6). Curve 7 gives the maximum possible value of the dimensionless temperature as $\mathrm{Pr} \to \infty$ ($\varkappa_s = 0$); the other parameters have the same value as in Fig. 3. At a fixed value of Pr, a decrease in the Reynolds number (equivalent to a decrease in the initial pore size) leads to a substantial decrease in the end (highest) temperature Θ_1, which

firms the conclusion concerning the effect of the size of the inhomogeneities on the conditions for formation of a local melting zone and the time required for thermal equilibrium to be established in the porous material.

For real materials the Prandtl number varies within wide limits, which is associated with the broad range of variation of the coefficient of viscosity and thermal diffusivity for different materials. Thus, for metals, assuming $\rho_s = 10^4$ kg/m^3, $\varkappa_s = 10^{-4}$-10^{-5} m^2/sec, and $\eta = 10^3$-10^4 Pa·sec, we obtain Pr = 10^3-10^5.

As the amplitude of the shock wave increases, so does the Reynolds number, whereas the β parameter decreases, which also leads to a decrease in the pore collapse time. Consequently, with increase in shock wave intensity the effect of the initial pore size on the heating dynamics will grow weaker or, more precisely, be shifted in the direction of lower values of a_0. From this it follows that the effect of the size of the inhomogeneities on the heating dynamics of a porous material under shock loading will be most strongly expressed in the region of pressures comparable with its strength, which is also confirmed by the results of experimental research [10-12, 15].

These investigations show that the nonequilibrium nature of the heating of the shocked material depends to a considerable extent not only on the initial porosity but also on the initial pore size. The highest temperatures are attained in the neighborhood of the structural inhomogeneities, and in the limiting cases of large and small Reynolds numbers the maximum possible temperature rise is determined only by the plastic properties of the medium. As a result of an analysis of the effect of thermal conduction on the temperature profile developing in the vicinity of a pore it has been found that the end (highest) value of the temperature and the time required for the establishment of thermal equilibrium in the shocked material depend to a considerable extent on the initial size of the inhomogeneities.

LITERATURE CITED

1. S. Z. Dunin and V. V. Surkov, "Structure of the shock front in a porous solid," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1979).
2. S. Z. Dunin and V. V. Surkov, "Dynamics of pore closing in a shock front," Zh. Prikl. Mekh. Tekh. Fiz., 43, No. 3 (1979).
3. V. S. Trofimov, G. A. Adadurov, et al., "Anomalous shock compressibility of porous materials," Fiz. Goreniya Vzryva, 4, No. 2 (1968).
4. J. H. Blackburn and L. B. Seely, "Source of the light recorded in photographs of shocked granular pressings," Nature, 194, No. 4826 (1962).
5. J. H. Blackburn and L. B. Seely, "Light emitted from shocked granular sodium chloride in a vacuum," Nature, 202, No. 4929 (1964).
6. V. F. Nesterenko, "Electrical effects associated with the shock loading of a metal–metal contact," Fiz. Goreniya Vzryva, 11, No. 3 (1975).
7. G. V. Belyakov, "Shock deformation of granular media," Dokl. Akad. Nauk SSSR, 218, No. 6 (1974).
8. G. V. Belyakov, L. D. Lifshits, and V. N. Rodionov, "Shock deformation of an inhomogeneous medium modeled by a set of steel balls," Izv. Akad. Nauk SSSR, Fizika Zemli, No. 10 (1974).
9. S. Z. Dunin and V. V. Surkov, "Energy dissipation effects and influence of melting on the shock compression of porous bodies," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1982).
10. V. F. Nesterenko, "Shock compression of multicomponent materials," in: Continuum Dynamics, No. 29. Mechanics of Explosive Processes [in Russian], IG Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1977).
11. G. V. Belyakov, V. N. Rodionov, and V. P. Samosadnyi, "Heating of a porous material in shock compression," Fiz. Goreniya Vzryva, 13, No. 4 (1977).
12. O. V. Roman, V. F. Nesterenko, and I. M. Pikus, "Effect of powder particle size on the explosive pressing process," Fiz. Goreniya Vzryva, 15, No. 5 (1979).
13. C. L. Shott, "Effect of particle size of shock initiation of PETN, RDX and TETRYL," in: Proc. 5th Symp. (Internat.) on Detonation, Office of Naval Research ACR-184 (1970).
14. B. C. Taylor and L. H. Ervin, "Separation of ignition and buildup to detonation in pressed TNT," in: Proc. 6th Symp. (Internat.) on Detonation, Office of Naval Research ACR-221 (1976).
15. P. Howe, R. Frey, B. Taylor, and V. Boyle, "Shock initiation and the critical energy concept," in: Proc. 6th Symp. (Internat.) on Detonation, Office of Naval Research ACR-221 (1976).

16. R. R. Boade, "Principal Hugoniot, second-shock Hugoniot and release behavior of pressed copper powder," J. Appl. Phys., 41, No. 11 (1970).
17. J. J. Bhatt, M. M. Carroll, and J. F. Shatz, "A spherical model calculation for volumetric response of porous rocks," Trans. ASME, Ser. E, J. Appl. Mech., 42, No. 2 (1975).
18. V. N. Mineev and E. V. Savinov, "Viscosity and melting point of aluminum, lead, and sodium chloride in shock compression," Zh. Eksp. Teor. Fiz., 52, No. 3 (1967).
19. G. V. Stepanov, "Coefficient of viscosity of metallic materials subjected to high-speed deformation in elastoplastic loading waves," in: Detonation. Critical Phenomenon. Physico-Chemical Transitions in Shock Waves [in Russian], Chernogolovka(1978).
20. G. I. Kanel', "Viscoelastic properties of metals in a shock wave," in: Detonation. Critical Phenomena. Physico-Chemical Transitions in Shock Waves [in Russian], Chernogolovka (1978).

FRACTURE WAVE IN A CHAIN STRUCTURE

L. I. Slepyan and L. V. Troyankina

UDC 534.1:539.375

Various formulations of the problem of fracture wave propagation in an elastic brittle body are known (see [1-4] including references). Each of the proposed variants of the theory of this process is based on some hypothesis, for example: concerning the fracture wave velocity [2-5], the intensity of the elastic precursor [6], or the fracture energy [7, 8]. The introduction of an additional relation is necessary in order to close the system of equations of dynamics of the elasto-brittle continuum. However, such a relation cannot be justified without having recourse to data on the structure of the fracture front. This distinguishes the fracture wave from "ordinary" nonlinear waves whose macroparameters are determined independently of the structure of the front [8].

This fundamental difficulty can be overcome if we consider a structured medium, as is done below. As the simplest model of a structured medium we will take a linear chain in which each of the component unit masses interacts with the two adjacent masses through linear-elastic inertialess bonds (the distance between masses and the stiffnesses of the bonds are also taken as units of measurement, the velocity of the long waves in the undamaged chain being taken as the unit of velocity). At a certain bond stress $\sigma = \sigma_* \ll 1$ the bond partially fails: The bond stiffness takes a (positive) value $\alpha^2 < 1$. As distinct from the formulation of the same problem within a continuum framework [5-8], here there is no need to introduce any additional hypotheses.

Let us consider the stationary problem, in solving which we will use the same methods as in investigating the dynamics of a crack in a grid [9]. In the problem in question, taking into account the structure leads, essentially, to the same result as in [9]: high-frequency waves carrying part of the energy away from the fracture front (effect analogous to a temperature rise [7]). In the dynamics of a single crack, the structure of the medium determines the macroscopic fracture criterion and, consequently, affects the macroparameters of the velocity and stress fields. Thus, in the propagation of a fracture wave the microstructure also determines the macroparameters of the process (longwave approximation) — the ratios σ_1/σ_*, σ_2/σ_*, where $\sigma_1 = \text{const}$ is the average stress in the elastic precursor, and $\sigma_2 = \text{const}$ is the average stress behind the fracture front. The earlier assumptions [8] to the effect that $\sigma_1 < \sigma_*$ and fracture can occur even when $\sigma_2 < \sigma_*$ ($\sigma_{1,2} > 0$) are confirmed. The latter conclusion might appear strange if considered within the context of an unstructured continuum. Here, however, it is obvious: The total stresses behind the fracture front (with the high-frequency waves taken into account) exceed the average value.

1. Formulation of the Problem and Basic Relations

We assume that the velocities u and accelerations a are functions of a single variable $\tau = x - vt$, where $x = 0, \pm 1, \ldots$ are the Lagrangian coordinates of the masses, $v = \text{const} > 0$ is the velocity of the fracture front, and t is time. We note that the displacements, which owing to the presence of the elastic precursor also depend on x, cannot be similarly defined.

Leningrad, Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 128-134, November-December, 1984. Original article submitted September 24, 1982.

0021-8944/84/2506- 0921$08.50